Special Practice Problems

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~[JEE (Mains & Advanced)]~

Topic: Continuity & Differentiability

**** सही दिशा में उठाया गया एक छोटा कदम भी बहुत बड़ा साबित होता है ****

Only One Option is Correct

Each question in this part has four choices out of which just one is correct. Indicate your choice of correct answer for each question by writing one of the letters a, b, c, d whichever is appropriate.

1. If the function $f(x)=\left[\frac{(x-2)^3}{a}\right]\sin(x-2)+a\cos(x-2)$, [.] denotes the greatest integer function, is continuous and differentiable in (4, 6), then

(a) $a\in[8,64]$ (b) $a\in(0,8]$

(c) $a\in[64,\infty)$ (d) none of these

2. The set of all points, where $f(x)=\sqrt[3]{x^2|x|}-|x|-1$ is not differentiable is

(a) {0} (b) {–1, 0, 1}

(c) {0, 1} (d) none of these

3. Let $f(x)=x^3+x$ be function and $g(x)=\begin{cases}f(|x|), & x\ge 0\\ f(-|x|), & x<0\end{cases}$, then

(a) $g(x)$ is continuous $\forall\ x\in R$

(b) $g(x)$ is continuous $\forall\ x\in R^-$ only

(c) $g(x)$ is continuous $\forall\ x\in R^+$ only

(d) $g(x)$ is discontinuous $\forall x\in R^-$

4. Which of the following is not continuous for all x ?

(a) $|x-1|+|x-2|$ (b) $x^2-|x-x^3|$

(c) $\sin|x|+|\sin x|$ (d) $\frac{\cos x}{|\cos x|}$

5. If $[x]$ denotes the integral part of x and $f(x)=[n+p\sin x]$, $0<x<\pi$, $n\in I$ and p is a prime number, then the number of points, where $f(x)$ is not differentiable is

(a) $p-1$ (b) p

(c) $2p-1$ (d) $2p+1$

6. If $f(x)=(x^2-4)|(x^3-6x^2+11x-6|+\frac{x}{1+|x|}$, then the set of points at which the function $f(x)$ is not differentiable is

(a) {– 2, 2, 1, 3} (b) {– 2, 0, 3}

(c) {– 2, 2, 0} (d) {1, 3}

7. If $f(x)=\{x^2\}-(\{x\})^2$, where $\{x\}$ denotes the fractional part of x, then

(a) $f(x)$ is continuous at $x=2$ but not at $x=-2$

(b) $f(x)$ is continuous at $x=-2$ but not at $x=2$

(c) $f(x)$ is continuous at $x=-2$ but not at $x=-2$

(d) $f(x)$ is discontinuous at $x=2$ and $x=-2$

8. Given, $f(x)=x^2e^{2(x-1)}$, $0\le x\le 1$

$= a\ Sgn\ (x+1)\cos(2x-2)+bx^2$, $1<x\le 2$,

$f(x)$ is differentiable at $x=1$ provided

(a) $a=-1, b=2$ (b) $a=1, b=-2$

(c) $a=-3, b=4$ (d) $a=3, b=-4$

9. If $f(x)=\begin{cases}\frac{\sin\{\cos x\}}{x-(\pi/2)}, & x\ne\frac{\pi}{2}\\ 1, & x=\frac{\pi}{2}\end{cases}$, where {.} represents the fractional part function, then $f(x)$ is :

(a) continuous at $x=\frac{\pi}{2}$

(b) $\lim_{x\to\frac{\pi}{2}} f(x)$ exists, but f is not continuous at $x=\frac{\pi}{2}$

(c) $\lim_{x\to\frac{\pi}{2}} f(x)$ does not exist

(d) $\lim_{x\to\frac{\pi}{2}^-} f(x)=-1$

10. If $f(x)=\frac{1}{(x-1)(x-2)}$ and $g(x)=\frac{1}{x^2}$, then points of discontinuity of $f(g(x))$ are

(a) $\left\{-1,0,1,\frac{1}{\sqrt2}\right\}$ (b) $\left\{-\frac{1}{\sqrt2},-1,0,1,\frac{1}{\sqrt2}\right\}$

(c) {0, 1} (d) $\left\{0,1,\frac{1}{\sqrt2}\right\}$

11. The value of the $f(0)$, so that the function $f(x)=\dfrac{\sqrt{(a^2-ax+x^2)}-\sqrt{(a^2+ax+x^2)}}{\sqrt{(a+x)}-\sqrt{(a-x)}}$ becomes continuous for all x, is given by

(a) $a\sqrt{a}$ (b) $\sqrt{a}$

(c) $-\sqrt{a}$ (d) $-a\sqrt{a}$

12. Let $f(x)=\tan(\pi/4-x)/\cot 2x\ (x\neq\pi/4)$. The value which should be assigned to f at $x=\pi/4$. So that it is continuous every where, is

(a) 1/2 (b) 1

(c) 2 (d) none of these

13. If $f(x)=\dfrac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2}\ (x\neq 0)$; then for f to be continuous every where, $f(0)$ is equal to

(a) -1 (b) 1

(c) 2^6 (d) none of these

14. Let f be a function satisfying $f(x+y)=f(x)+f(y)$ and $f(x)=x^2g(x)$ for all x and y, where $g(x)$ is a continuous function, then $f'(x)$ is equal to

(a) $g'(x)$ (b) $g(0)$

(c) $g(0)+g'(x)$ (d) 0

15. Let $f(x+y)=f(x)f(y)$ for all x and y. Suppose that $f(3)=3$ and $f'(0)=11$, then $f'(3)$ is given by

(a) 22 (b) 44

(c) 28 (d) none of these

16. Let $f:R\to R$ be a differentiable function and $f(1)=4$. Then the value of $\lim\limits_{x\to 1}\int_4^{f(x)}\dfrac{2t}{x-1}dt$ is

(a) $8f'(1)$ (b) $4f'(1)$

(c) $2f'(1)$ (d) $f'(1)$

17. Let $f:R\to R$ be a function such that $f\left(\dfrac{x+y}{3}\right)=\dfrac{f(x)+f(y)}{3}$, $f(0)=3$ and $f'(0)=3$, then

(a) $\dfrac{f(x)}{x}$ is differentiable in R

(b) $f(x)$ is continuous but not differentiable in R

(c) $f(x)$ is continuous in R

(d) $f(x)$ is bounded in R

18. If $f(x)=\dfrac{\tan\pi[(2x-3\pi)^3]}{1+[2x-3\pi]^2}$ ([.] denotes the greatest integer function), then

(a) $f(x)$ is continuous in R

(b) $f(x)$ is continuous in R but not differentiable in R

(c) $f'(x)$ exists everywhere but $f'(x)$ does not exist at some $x\in R$

(d) none of the above

19. If $f(x)=\begin{cases}1, & x \text{ is rational}\\ 2, & x \text{ is irrational}\end{cases}$, then

(a) $f(x)$ is continuous in $R\sim I$

(b) $f(x)$ is continuous in $R\sim Q$

(c) $f(x)$ is continuous in R but not differentiable in R

(d) $f(x)$ is neither continuous nor differentiable in R

20. If $f(x)$ is a twice differentiable function, then between two consecutive roots of the equation $f'(x)=0$, there exists

(a) at least one root of $f(x)=0$

(b) at most one root of $f(x)=0$

(c) exactly one root of $f(x)=0$

(d) at most one root of $f''(x)=0$

21. If the derivative of the function $f(x)=\begin{cases}bx^2+ax+4; & x\geq -1\\ ax^2+b; & x<-1\end{cases}$, is everywhere continuous, then

(a) $a=2, b=3$ (b) $a=3, b=2$

(c) $a=-2, b=-3$ (d) $a=-3, b=-2$

22. If $f(x)=\begin{cases}\sin x, & x\neq n\pi, n\in I\\ 2, & \text{otherwise}\end{cases}$ and

$g(x)=\begin{cases}x^2+1, & x\neq 0,2\\ 4, & x=0\\ 5, & x=2\end{cases}$, then $\lim\limits_{x\to 0} g\{f(x)\}$ is

(a) 5 (b) 6

(c) 7 (d) 1

23. The function $f(x)=|2\,Sgn\,2x|+2$ has

(a) jump discontinuity

(b) removal discontinuity

(c) infinite discontinuity

(d) no discontinuity at $x=0$

24. If the function

$f(x)=\begin{cases}(1+|\sin x|)^{a/|\sin x|}, & -\dfrac{\pi}{6}<x<0\\ b, & x=0\\ e^{\tan 2x/\tan 3x}, & 0<x<\dfrac{\pi}{6}\end{cases}$, is continuous at $x=0$, then

(a) $a=\log_e b, a=2/3$ (b) $b=\log_e a, a=2/3$

(c) $a=\log_e b, b=2$ (d) none of these

25. If $f(x)=\int_{-1}^{x}|t|\,dt, x\geq -1$, then

(a) f and f' are continuous for $x+1>0$

(b) f is continuous but f' is not so for $x+1>0$

(c) f and f' are continuous at $x=0$

(d) f is continuous at $x=0$ but f' is not so

26. Let $f(x)=\begin{cases}x^n\sin\left(\dfrac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$, then $f(x)$ is continuous but not differentiable at $x=0$ if

(a) $n\in(0,1]$ (b) $n\in[1,\infty)$

(c) $n\in(-\infty,0)$ (d) $n=0$

27. Let [·] represents the greatest integer function and $f(x)=[\tan^2 x]$, then

(a) $\lim\limits_{x\to 0}f(x)$ does not exist

(b) $f(x)$ is continuous at $x=0$

(c) $f(x)$ is non-differentiable at $x=0$

(d) $f'(0)=1$

28. $f(x)=\begin{cases} x-[x], \text{ for } 2n\le x<2n+1, n\in N, \\ \text{where}[x]=\text{Integral part of } x\le x \\ \frac{1}{2}, \text{ for } 2n+1\le x<2n+2 \end{cases}$

the function
(a) is discontinuous at $x=1,2$
(b) is periodic with period 1
(c) is periodic with period 2
(d) $\int_0^2 f(x)\,dx$ exists

29. A function, $f(x)=x\left[1+\frac{1}{3}\sin(\log x^2)\right], x\ne 0$

$f(0)=0$

[·] = Integral part, the function
(a) is continuous at $x=0$
(b) is monotonic
(c) is derivable at $x=0$
(d) cannot be defined for $x<-1$

30. If $f(x)=\begin{cases}[\cos \pi x], & x<1 \\ |x-2|, & 1\le x<2\end{cases}$

([·] denotes the greatest integer function), then $f(x)$ is
(a) continuous and non-differentiable at $x=-1$ and $x=1$
(b) continuous and differentiable at $x=0$
(c) discontinuous at $x=1/2$
(d) continuous but not differentiable at $x=2$

31. If $f(x)=[\sqrt{2}\sin x]$, where $[x]$ represents the greatest integer function $\le x$, then
(a) $f(x)$ is periodic
(b) maximum value of $f(x)$ is 1 in the interval $[-2\pi, 2\pi]$
(c) $f(x)$ is discontinuous at $x=\frac{n\pi}{2}+\frac{\pi}{4}, n\in I$
(d) $f(x)$ is differentiable at $x=n\pi, n\in I$

32. The function defined by $f(x)=(-1)^{[x^3]}$([·] denotes greatest integer function) satisfies
(a) discontinuous for $x=n^{1/3}$, where n is any integer
(b) $f(3/2)=1$
(c) $f'(x)=0$ for $-1<x<1$
(d) none of the above

33. If $f(x)$ be a continuous function defined for $1\le x\le 3$, $f(x)\in Q\,\forall x\in[1,3]$, $f(2)=10$, (where Q is a set of all rational numbers) then $f(1.8)$ is
(a) 1
(b) 5
(c) 10
(d) 20

34. Let $f(x)=\begin{cases} x^p\sin\left(\frac{1}{x}\right)+x|x|, & x\ne 0 \\ 0, & x=0 \end{cases}$ the set of values, of p for which $f''(x)$ is everywhere continuous is
(a) $[2,\infty)$
(b) $[-3,\infty)$
(c) $[5,\infty)$
(d) none of these

35. The value of p for which the function

$f(x)=\dfrac{(4^x-1)^3}{\sin(x/p)\ln\left(1+\frac{x^2}{3}\right)}, x\ne 0$

$=12(\ln 4)^3, x=0$

may be continuous at $x=0$ is
(a) 1
(b) 2
(c) 3
(d) 4

36. The value of $f(0)$, so that the function $f(x)=\dfrac{1-\cos(1-\cos x)}{x^4}$ is continuous everywhere is
(a) 1/8
(b) 1/2
(c) 1/4
(d) none of these

37. The jump of the function at the point of the discontinuity of the function $f(x)=\dfrac{1-k^{1/x}}{1+k^{1/x}}$ $(k>0)$ is
(a) 4
(b) 2
(c) 3
(d) none of these

38. Let $f''(x)$ be continuous at $x=0$ and $f''(0)=4$. The value of $\lim\limits_{x\to 0}\dfrac{2f(x)-3f(2x)+f(4x)}{x^2}$ is
(a) 6
(b) 10
(c) 11
(d) 12

39. Let $f(x)=[x]+\sqrt{x-[x]}$, where $[x]$ denotes the greatest integer function. Then
(a) $f(x)$ is continuous on R^+
(b) $f(x)$ is continuous on R
(c) $f(x)$ is continuous on $R\sim I$
(d) none of the above

40. Let $f(x)=\begin{cases}\dfrac{a|x^2-x-2|}{2+x-x^2}, & x<2 \\ b, & x=2 \\ \dfrac{x-[x]}{x-2}, & x>2\end{cases}$

([.] denotes the greatest integer function)
If $f(x)$ is continuous at $x=2$, then
(a) $a=1, b=2$
(b) $a=1, b=1$
(c) $a=0, b=1$
(d) $a=2, b=1$

41. The function $f(x)=\dfrac{e^{\tan x}-1}{e^{\tan x}+1}$ is discontinuous at x is equal to
(a) $n\pi+\pi, n\in I$
(b) $n\pi+\pi/2, n\in I$
(c) $n\pi+\pi/4, n\in I$
(d) $n\pi+\pi/8, n\in I$

42. Let $f(x)=\begin{cases}-1, & x<0 \\ 0, & x=0 \\ 1, & x>0\end{cases}$ and $g(x)=\sin x+\cos x$, then points of discontinuity of $f\{g(x)\}$ in $(0,2\pi)$ is
(a) $\left\{\frac{\pi}{2},\frac{3\pi}{4}\right\}$
(b) $\left\{\frac{3\pi}{4},\frac{7\pi}{4}\right\}$
(c) $\left\{\frac{2\pi}{3},\frac{5\pi}{3}\right\}$
(d) $\left\{\frac{5\pi}{4},\frac{7\pi}{3}\right\}$

43. The points of discontinuity of the function

$f(x) = \lim_{n\to\infty} \dfrac{(2\sin x)^{2n}}{3^n - (2\cos x)^{2n}}$ are given by

(a) $r\pi \pm \dfrac{\pi}{12}, r \in I$ (b) $r\pi \pm \dfrac{\pi}{6}, r \in I$

(c) $r\pi \pm \dfrac{\pi}{3}, r \in I$ (d) none of these

44. Let $f(x) = [\cos x + \sin x], 0 < x < 2\pi$, where $[x]$ denotes the greatest integer less than or equal to x. The number of points of discontinuity of $f(x)$ is

(a) 6 (b) 5

(c) 4 (d) 3

45. The function $f(x) = |x^2 - 3x + 2| + \cos|x|$ is not differentiable at x is equal to

(a) -1 (b) 0

(c) 1 (d) 2

46. Let $f(x) = \dfrac{1 - \tan x}{4x - \pi}, x \neq \pi/4$ and $x \in [0, \pi/2)$

$= \lambda, x = \pi/4$

If $f(x)$ is continuous in $(0, \pi/2]$, then λ is

(a) 1 (b) 1/2

(c) $-1/2$ (d) none of these

47. If $g(x) = \begin{cases} [f(x)], & x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \\ 3, & x = \pi/2 \end{cases}$

where $[x]$ denotes the greatest integer function and

$f(x) = \dfrac{2(\sin x - \sin^n x) + |\sin x - \sin^n x|}{2(\sin x - \sin^n x) - |\sin x - \sin^n x|}, n \in R$, then

(a) $g(x)$ is continuous and differentiable at $x = \pi/2$, when $0 < n < 1$

(b) $g(x)$ is continuous and differentiable at $x = \pi/2$, when $n > 1$

(c) $g(x)$ is continuous but not differentiable at $x = \pi/2$, when $0 < n < 1$

(d) $g(x)$ is continuous but not differentiable, at $x = \pi/2$, when $n > 1$

48. $f(x) = 1 + x(\sin x)[\cos x], 0 < x \le \pi/2$

([.] denotes the greatest integer function)

(a) is continuous in $(0, \pi/2)$

(b) is strictly decreasing in $(0, \pi/2)$

(c) is strictly increasing in $(0, \pi/2)$

(d) has global maximum value 2

49. If $f(x)$ is a continuous function $\forall x \in R$ and the range of $f(x)$ is $(2, \sqrt{26})$ and $g(x) = \left[\dfrac{f(x)}{c}\right]$ is continuous $\forall x \in R$, then the least positive integral value of c is (where [.] denotes the greatest integer function)

(a) 2 (b) 3

(c) 5 (d) 6

50. If $f(x) = \begin{cases} a + \dfrac{\sin[x]}{x}, & x > 0 \\ 2, & x = 0 \\ b + \left[\dfrac{\sin x - x}{x^3}\right], & x < 0 \end{cases}$

(where [.] denotes the greatest integer function). If $f(x)$ is continuous at $x = 0$, then b is equal to

(a) $a - 2$ (b) $a - 1$

(c) $a + 1$ (d) $a + 2$

51. Which of the following functions are differentiable in $(-1, 2)$?

(a) $\int_x^{2x} (\log x)^2 dx$ (b) $\int_x^{2x} \dfrac{\sin x}{x} dx$

(c) $\int_0^x \dfrac{1 - t + t^2}{1 + t + t^2} dt$ (d) none of these

52. Let $f(x) = \begin{cases} (1 + |\cos x|)^{ab/|\cos x|}, & n\pi < x < (2n+1)\pi/2 \\ e^a \cdot e^b, & x = (2n+1)\pi/2 \\ e^{\cot 2x/\cot 8x}, & (2n+1)\pi/2 < x < (n+1)\pi \end{cases}$

If $f(x)$ is continuous in $(n\pi, (n+1)\pi)$, then

(a) $a = 1, b = 2$ (b) $a = 2, b = 2$

(c) $a = 2, b = 3$ (d) $a = 3, b = 4$

More Than One Options are Correct

Each question in this part has more than one correct answers. For each question, write the letters a, b, c, d corresponding to the correct answers.

1. Let $f(x) = \dfrac{1}{[\sin x]}$, ([.] denotes the greatest integer function) then

(a) domain of $f(x)$ is $(2n\pi + \pi, 2n\pi + 2\pi) \cup \{2n\pi + \pi/2\}$, where $n \in I$

(b) $f(x)$ is continuous, when $x \in (2n\pi + \pi, 2n\pi + 2\pi)$, where $n \in I$

(c) $f(x)$ is differentiable at $x = \pi/2$

(d) none of the above

2. Let $g(t) = [t(1/t)]$ for $t > 0$ ([.] denotes the greatest integer function), then $g(g)$ has :

(a) discontinuities at finite number of points

(b) discontinuities at infinite number of points

(c) $g(1/2) = 1$

(d) $g(3/4) = 1$

3. Let $f(x)$ and $\phi(x)$ be defined by $f(x) = [x]$ and $\phi(x) = \begin{cases} 0, & x \in I \\ x^2, & x \in R - I \end{cases}$ (where [.] denotes the greatest integer function), then

(a) $\lim_{x\to 1} \phi(x)$ exists, but ϕ is not continuous at $x = 1$

(b) $\lim_{x \to 1} f(x)$ does not exist and f is not continuous at $x = 1$
(c) ϕ *of* continuous for all x
(d) $f o \phi$ is continuous for all x

4. The following functions are continuous on $(0, \pi)$:
(a) $\tan x$
(b) $\int_0^{\pi} t \sin\left(\frac{1}{t}\right) dt$
(c) $\begin{cases} 1, & 0 < x \le \frac{3\pi}{4} \\ 2\sin\frac{2}{9}x, & \frac{3\pi}{4} < x < \pi \end{cases}$
(d) $\begin{cases} x \sin x, & 0 < x \le \pi/2 \\ \frac{\pi}{2}\sin(\pi + x), & \frac{\pi}{2} < x < \pi \end{cases}$

5. If $f(x) = \frac{\tan([x]\pi)}{[1 + |\ln(\sin^2 x + 1)|]}$, where [.] denotes the greatest integer function, then $f(x)$ is :
(a) continuous $\forall\ x \in R$
(b) discontinuous $\forall\ x \in I$
(c) non-differentiable $\forall\ x \in I$
(d) a periodic function with fundamental period not defined

6. If $f'(x) = g(x)(x - a)^2$, where $g(a) \ne 0$ and g is continuous at $x = a$, then :
(a) f is increasing near a if $g(a) > 0$
(b) f is increasing near a if $g(a) < 0$
(c) f is decreasing near a if $g(a) > 0$
(d) f is decreasing near a if $g(a) < 0$

7. A function, which is continuous and not differentiable at the origin, is :
(a) $f(x) = x$ for $x < 0$ and $f(x) = x^2$ for $x \ge 0$
(b) $g(x) = x$ for $x < 0$ and $g(x) = 2x$ for $x \ge 0$
(c) $h(x) = x|x|, x \in R$
(d) $k(x) = 1 + |x|, x \in R$

8. Let $f(x) = \lim_{n \to \infty} (\sin x)^{2n}$, then f is :
(a) continuous at $x = \pi/2$
(b) discontinuous at $x = \pi/2$
(c) discontinuous at $x = -\pi/2$
(d) discontinuous at an infinite number of points

9. If $f(x) = \tan^{-1} \cot x$, then :
(a) $f(x)$ is periodic with period π
(b) $f(x)$ is discontinuous at $x = \pi/2, 3\pi/2$
(c) $f(x)$ is not differentiable at $x = \pi, 99\pi, 100\pi$
(d) $f(x) = -1$, for $2n\pi \le x \le (2n+1)\pi$

10. Let $f(x) = \begin{cases} \int_0^x (1 + |1 - t|)\, dt, & x > 2 \\ 5x + 1, & x \le 2 \end{cases}$,
then
(a) $f(x)$ is not continuous at $x = 2$
(b) $f(x)$ is continuous but not differentiable at $x = 2$
(c) $f(x)$ is differentiable everywhere
(d) the right derivative of $f(x)$ at $x = 2$ does not exist

11. $f(x) = \min\{1, \cos x, 1 - \sin x\}, -\pi \le x \le \pi$, then
(a) $f(x)$ is not differentiable at 0
(b) $f(x)$ is differentiable at $\frac{\pi}{2}$
(c) $f(x)$ has local maxima at 0
(d) none of the above

12. If $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1 + x^2)}, & x \ne 0 \\ 0, & x = 0 \end{cases}$, then
(a) f is continuous at $x = 0$
(b) f is continuous at $x = 0$ but not differentiable at $x = 0$
(c) f is differentiable at $x = 0$
(d) f is not continuous at $x = 0$

13. Let $f(x) = \frac{\sin(\pi[x - \pi])}{1 + [x^2]}$, where [.] denotes the greatest integer function. Then $f(x)$ is
(a) continuous at integral points
(b) continuous everywhere but not differentiable
(c) differentiable once but $f'(x), f''(x), \ldots$ do not exist
(d) differentiable for all x

14. The function $f(x) = \begin{cases} |2x - 3|[x]; & x \ge 1 \\ \sin\left(\frac{\pi x}{2}\right); & x < 1 \end{cases}$
([.]) denotes the greatest integer function)
(a) is conerentiable at $x = 0$
(c) is continuous at $x = 0$
(b) is difftinuous but not differentiable at $x = 1$
(d) is continuous but not differentiable at $x = 3/2$

15. Let $h(x) = \min\{x, x^2\}$ for every real number x. Then
(a) h is continuous for all x
(b) h is differentiable for all x
(c) $h'(x) = 1$ for all $x > 1$
(d) h is not differentiable at two values of x

16. A function $f(x)$ is defined in the interval [1, 4] as follows :
$f(x) = \begin{cases} \log_e [x], & 1 \le x < 3 \\ |\log_e x|, & 3 \le x < 4 \end{cases}$
the graph of the function $f(x)$
(a) is broken at two points
(b) is broken at exactly one point
(c) does not have a definite tangent at two points
(d) does not have a definite tangent at more than two points

17. If $f(x) = \min(\tan x, \cot x)$, then :
(a) $f(x)$ is discontinuous at $x = 0, \frac{\pi}{4}, \frac{5\pi}{4}$
(b) $f(x)$ is continuous at $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$
(c) $\int_0^{\pi/2} f(x)\, dx = 2 \ln \sqrt{2}$
(d) $f(x)$ is periodic with period π

18. $f(x) = \frac{[1/2 + x] - [1/2]}{x}, -1 \le x \le 2$
has ([.] denotes the greatest integer function)

(a) discontinuity at $x = 0$ (b) discontinuity at $x = 1/2$
(c) discontinuity at $x = 1$ (d) discontinuity at $x = 3/2$

19. If $f(x) = [x] + [x + 1/3] + [x + 2/3]$, then ([.] denotes the greatest integer function)
(a) $f(x)$ is discontinuous at $x = 1, 10, 15$
(b) $f(x)$ is continuous at $x = n/3$, where n is any integer
(c) $\int_0^{2/3} f(x)\,dx = 1/3$
(d) $\lim_{x \to 2/3} f(x) = 2$

20. If $f(x) = \begin{cases} (\sin^{-1} x)^2 \cdot \cos(1/x), & x \ne 0 \\ 0, & x = 0 \end{cases}$, then
(a) $f(x)$ is continuous everywhere in $x \in [-1, 1]$
(b) $f(x)$ is continuous no where in $x \in [-1, 1]$
(c) $f(x)$ is differentiable everywhere in $x \in (-1, 1)$
(d) $f(x)$ is differentiable no where in $x \in [-1, 1]$

21. Give, a real valued function 'f such that
$$f(x) = \begin{cases} \frac{\tan^2 x}{(x^2 - [x])^2} & \text{for } x > 0 \\ 1 & \text{for } x = 0 \\ \sqrt{\{x\} \cot \{x\}} & \text{for } x < 0 \end{cases}$$
where, $[x]$ is the integral part and $\{x\}$ is the fractional part of x, then
(a) $\lim_{x \to 0} f(x) = 1$ (b) $\lim_{x \to 0^-} f(x) = \cot 1$
(c) $\cot^{-1}\left(\lim_{x \to 0^-} f(x)\right)^2 = 1$ (d) f is continuous at $x = 0$

22. The function $f(x) = \left[x^2 \left[\frac{1}{x^2}\right]\right], x \ne 0$ is ($[x]$ represents the greatest integer $\le x$)
(a) continuous at $x = 1$
(b) discontinuous at $x = -1$
(c) discontinuous at infinitely many points
(d) continuous everywhere

Linked Comprehension Type Questions

*In this section each paragraph has some multiple choice questions have to be answered. Each question has four choices a, b, c, d out of which **Only One** is correct.*

PASSAGE 1

$$\text{Let } f(x) = \begin{cases} \frac{a(1 - x\sin x) + b\cos x + 5}{x^2} & , x < 0 \\ 3 & , x = 0 \\ \left\{1 + \left(\frac{cx + dx^3}{x^2}\right)\right\}^{1/x} & , x > 0 \end{cases}$$

If f is continuous at $x = 0$

On the basis of above information, answer the following questions:

1. The value of a is
(a) -1 (b) ln 3
(c) 0 (d) -4

2. The value of b is
(a) -1 (b) ln 3
(c) 0 (d) -4

3. The value of c is
(a) 2 (b) 3
(c) 0 (d) None of these

4. The value of e^d is
(a) 0 (b) 1
(c) 2 (d) 3

5. The minimum value of $f(x)$ is
(a) 1 (b) 2
(c) 3 (d) none of these

PASSAGE 2

Let $f: R \to R$ be a differential function satisfying $f\left(\frac{x+y}{3}\right) = \frac{2 + f(x) + f(y)}{3}$ for all real x and y and $f'(2) = 2$.

On the basis of above information, answer the following questions :

1. The value of $f(1)$ is
(a) 0 (b) 2
(c) 4 (d) none of these

2. The range of $f(|x|)$ is
(a) $[0, \infty)$ (b) $[1, \infty)$
(c) $[2, \infty)$ (d) none of these

3. If $g(x) = |f(|x|) - 3|$ for all $x \in R$, then for $g(x)$
   (a) one non-differentiable point
   (b) two non-differentiable point
   (c) three non-differentiable point
   (d) four non-differentiable point
4. If $x \in [-2, 3]$, then range of $f(x)$ is
   (a) $[-2, 3]$ (b) $[-3, 4]$
   (c) $[-2, 8]$ (d) $[-3, 10]$
5. The number of solutions of the equation $x^2 + (f(|x|))^2 = 9$ are
   (a) 0 (b) 2
   (c) 3 (d) 5

PASSAGE 3

Let $f(x)$ be a real valued function not identically zero such that

$$f(x + y^n) = f(x) + \{f(y)\}^n \forall\ x, y \in R$$

Where, n is odd natural number > 1 and $f'(0) \geq 0$

On the basis of above information, answer the following questions :

1. The value of $f(2)$ is
   (a) 0 (b) -1
   (c) 2 (d) none of these
2. The value of $f(5)$ is
   (a) -5 (b) 0
   (c) 5 (d) not defined
3. The value of $f'(12)$ is
   (a) 1 (b) 0
   (c) 12 (d) 5
4. The function $f(x)$ is
   (a) even (b) odd
   (c) neither even nor odd (d) none of these
5. The function $f(x)$ is
   (a) discontinuous at one point
   (b) continuous everywhere but non-differentiable at some points
   (c) discontinuous at three points
   (d) continuous and differentiable everywhere

PASSAGE 4

Let
$$f(x) = \begin{cases} x + a, & x < 0 \\ |x - 1|, & x \geq 0 \end{cases}$$

and
$$g(x) = \begin{cases} x + 1, & x < 0 \\ (x-1)^2 + b, & x \geq 0 \end{cases}$$

where a and b are non-negative real numbers.

On the basis of above information, answer the following questions :

1. The value of a, if $(gof)x$ is continuous for all real x, is
   (a) -1 (b) 0
   (c) 1 (d) 2
2. The value of b, if $(gof)\,x$ is continuous for all real x, is
   (a) -1 (b) 0
   (c) 1 (d) 2
3. For these values of a and b, $(gof)x$ is
   (a) $\begin{cases} x + 2, & x < -1 \\ (x-2)^2, & -1 \leq x < 1 \\ x^2, & x \geq 1 \end{cases}$
   (b) $\begin{cases} x^2, & x < -1 \\ (x-2)^2, & -1 \leq x < 1 \\ x + 2, & x \geq 1 \end{cases}$
   (c) $\begin{cases} x + 2, & x < -1 \\ x^2, & -1 \leq x < 1 \\ (x-2)^2, & x \geq 1 \end{cases}$
   (d) $\begin{cases} (x-2)^2, & x < -1 \\ x^2, & -1 \leq x < 1 \\ x + 2, & x \geq 1 \end{cases}$
4. For these values of a and b, $(gof)x\ \forall x \in (-1, 1)$ is
   (a) even (b) odd
   (c) neither even nor odd (d) none of these
5. For these values of a and b, $(gof)x$ is
   (a) differentiable at $x = -1$
   (b) differentiable at $x = 0$
   (c) differentiable at $x = 1$
   (d) non-differentiable at $x = 2$